



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 12

Features of Option Embedded Bonds

J P Singh

Department of Management Studies



EFFECT OF CHANGES IN THE SHAPE OF THE YIELD CURVE ON THE VALUE OF A CALLABLE OR PUTTABLE BOND.



UPWARD SLOPING YIELD CURVE

- Let:
- $S_{01} = 3\%$;
- $S_{02} = 4.02\%$;
- $S_{03} = 5.069\%$



- The corresponding implied arbitrage free one-year forward rates:

$$f_{01} = S_{01} = 3\%$$

$$f_{12} = \frac{(1 + S_{02})^2}{(1 + S_{01})} - 1 = \frac{(1 + 0.0402)^2}{(1 + 0.03)} - 1$$
$$= 5.05\%$$

$$f_{23} = \frac{(1 + S_{03})^3}{(1 + S_{02})^2} - 1 = \frac{(1 + 0.05069)^3}{(1 + 0.0402)^2} - 1$$
$$= 7.20\%$$

INFERENCE

- **When the yield curve is upward sloping (i.e., normal), the more distant one-period forward rates are higher than the one-period forward rates in the near future.**



UPWARD SLOPING YIELD CURVE & VALUE OF OPTION BONDS

- The value of an embedded call option decreases as interest rates increase because higher interest rates lower the price of the straight bond and hence, reduce the probability of exercise of the call.



- **When the yield curve is upward sloping (i.e., normal), the more distant one-period forward rates are higher than the one-period forward rates in the near future.**
- **Because a higher interest rate scenario reduces the probability of the call option being in the money, the value of a call option will be lower for an upward sloping yield curve.**
- **As an upward-sloping yield curve becomes flatter, the call option value increases.**

- **The value of a put option increases with interest rates.**
- **When the yield curve is upward sloping, the probability of the put option going in the money is higher.**
- **Put option value therefore declines as an upward-sloping yield curve flattens.**

TERM STRUCTURE OF INTEREST RATES



TERM STRUCTURE

- The term structure of interest rates refers to the yields or interest rates **on similar securities** with different maturities (terms).

TYPES OF YIELD CURVES

- **Spot yield curve**
- **Par yield curve**
- **Forward yield curve**



SPOT YIELD CURVE



SPOT YIELD CURVES

- This yield curve shows spot rates by maturity.
- Spot rates are yields to maturity on zero-coupon bonds.

(Definition of YTM)

$$P_0 = \frac{F}{(1+y)^T}$$

Arbitrage Free Pricing

$$P_0 = \frac{F}{(1 + S_{0T})^T}$$
$$S_{0T} = y$$

- Hence, the spot rate yield curve (spot curve) depicts the YTM's of ZCBs of similar risk and of different maturities.



SPOT RATE:

RATIONALE OF DEFINITION

Spot interest rates of a particular maturity are the YTM's on zero coupon bonds of bonds of the same maturity.

$$P_0 = \frac{F}{(1 + S_{0T})^T} \quad (\text{DCF MODEL})$$

$$P_0 = \frac{F}{(1 + y)^T} \quad (\text{DEF of YTM}) \Rightarrow S_{0T} = y$$



IMPORTANT

- YTM OF ZCBs IS DETERMINED SOLELY BY MATURITY & PRICE
- YTMS FOR EACH MATURITY ARE DETERMINED BY USING THE PRICE AND A CURVE IS PLOTTED BETWEEN YTM & MATURITY TO GET THE SPOT CURVE.

$$P_0 = \frac{F}{(1 + y)^T}$$

$$P_0 = \frac{F}{(1 + S_{0T})^T}$$
$$S_{0T} = y$$

ZERO CURVE OR STRIP CURVE

- The spot yield curve is also called zero curve (for zero-coupon) or strip curve (because zero-coupon bonds are also called stripped bonds).



COUPON BONDS YIELD CURVE



WHAT IS YTM?

- YTM is the discount rate that equates the present value of future cash flows from the instrument to its current market price (including accrued interest).

$$P_0 = \sum_{t=1}^T \frac{C_t}{(1+y)^t}$$

OBSERVATION

- For a given price, the YTM depends on relative distribution of cash flows.
- In other words, YTM depends on:
 - the magnitude of the cash flows and
 - the timing of the cash flows.



YTM & RISKINESS

- For a given bond, YTM depends on the riskiness of the cash flows since the price will incorporate the riskiness in the realization of these cash flows.
- YTM is a risk adjusted discount rate that encapsulates the market's perception of the riskiness of the realizability of the cash flows from the instrument.

- **Therefore, two bonds with identical cash flows may trade at different YTM's if their risk as perceived by the market is different because the price will adjust accordingly.**
- **The more risky bond would quote at a lower price and hence a higher YTM.**

CONVERSE

- If two bonds carry similar risk perceptions in the market, they would have the same YTM notwithstanding that their cash flow patterns are different.
- The prices would adjust to make the YTMs equal.